CMSC 303.
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Theory of computation.

↳ study "nature" of computation

↳ physics : atom.

- Build models of computing

① Finite automata / regular expressions - pattern matching, text searching, hard ware design.

② Pushdown automata / context-free grammar - used in parsing, compilers

③ Turing machine - we believe captures all computing.

a) MATH BACKGROUND.

- set - a group of objects represented as a unit → CAN BE INFINITE/FINITE

e.g. $S = \{1, 2, 3, \ldots, a, z,$  $, \ldots\}$ ⟶ a collection of objects

↳ ① ORDER DOES NOT MATTER

$\{1, 2, 3\} = \{3, 2, 1\} \Rightarrow$ same set.

↳ ② REPEATED ELEMENTS DO NOT MATTER.

$\{1, 2, 2, 3\} = \{1, 2, 3\}$

- MEMBERSHIP

- when element is part of a set. ($5 \in S \Rightarrow$ 5 is in set S.)

- subset

$A \subseteq B$ (A is a subset of B)

↳ if $x \in A \Rightarrow x \in B$

(A is a strict subset of B) [proper]

$A \subset B$ if $A \subseteq B$, but $A \neq B$. $\Rightarrow$ e.g. $A = \{1, 2\}$ $B = \{1, 2, 3\}$ then $A \subset B$.

- EQUALITY

$A = B$ iff (if & only if) $A \subseteq B$ AND $B \subseteq A$.

example : $A = \{1, 3, 5\}$
$B = \{3, 1, 1, 5\}$ $\Rightarrow$ $A = B$ ✓

- IMPORTANT SETS

- Natural numbers, $\mathbb{N} = \{1, 2, 3, \ldots\}$
- integers, $\mathbb{Z} = \{\ldots -2, -1, 0, 1, 2 \ldots\}$.
- Empty set, $\emptyset = \{\}$

- UNION, $A \cup B := \{x \mid x \in A \text{ or } x \in B\}$

example : $A = \{1, 3\}$, $B = \{2, 4, 3\}$

$A \cup B = \{1, 2, 3, 4\}$

- INTERSECTION, $A \cap B := \{x \mid x \in A \text{ and } x \in B\}$
  example: $A \cap B := \{3\}$
- COMPLEMENT : requires notion of universe of elements.
  let Universe $U = \{1, 2, 3, 4\}$
  For set A, define $\bar{A} := \{x \mid x \notin A \text{ and } x \in U\}$
  example : $A = \{1\}$ ⇒ singleton
  $\bar{A} = \{2, 3, 4\}$
- SIZE = CARDINALITY : $|S|$ denotes the size of S
  example: $S = \{1, 2, 3\}$, $|S| = 3$ $S = \{1, 2, 3, 3\}$, $|S| = 3$. $S = \{\} = \emptyset$, $|S| = 0$
- POWER SETS ⇒ find all set of subsets of A.
  example. $A = \{0, 1\}$
  $P(A) = \{\emptyset, \{0\}, \{0,1\}, \{1\}\}$
  CLAIM: $|P(A)| = 2^{|A|}$
  PROOF SKETCH: 0 1 0 0 1 ⇔ $\{2, 5\}$
  suppose : $|A| = 5$
  e.g. $A\{1, 2, 3, 4, 5\}$

SEQUENCES & TUPLES
- SEQUENCE
  ① ORDER MATTERS & REPITION COUNTS
  example: $(-1, 5, 1) \neq (5, -1, 1) \neq (5, 5, -1, 1)$
  ↳ k-tuple - sequence of size k (k is finite)
- CARTESIAN PRODUCT / CROSS PRODUCT : $A \times B = \{(a, b) \mid a \in B \text{ and } b \in B\}$
  ↳ $A^k := \underbrace{A \times A \times A \ldots \times A}_{k\text{-times}}$ k-tuples from A
  example : $A = \{0, 1\}$
  $B = \{a, b\}$
  $A \times B = \{(0,a), (0,b), (1,a), (1,b)\}$ 2-tuple
  $|A \times B| = |A| \cdot |B|$

  $A \times B \times C$ ⇒ 3-tuples